NAG Fortran Library Routine Document

E01AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

E01AAF interpolates at a given point x from a table of function values y_i evaluated at equidistant or non-equidistant points x_i, for $i = 1, 2, \ldots, n+1$, using Aitken's technique of successive linear interpolations.

2 Specification

```
SUBROUTINE E01AAF(A, B, C, N1, N2, N, X)
INTEGER            N1, N2, N
real               A(N1), B(N1), C(N2), X
```

3 Description

This routine interpolates at a given point x from a table of values x_i and y_i, for $i = 1, 2, \ldots, n+1$ using Aitken's method. The intermediate values of linear interpolations are stored to enable an estimate of the accuracy of the results to be made.

4 References

Fröberg C E (1970) *Introduction to Numerical Analysis* Addison-Wesley

5 Parameters

1: A(N1) – ***real*** array *Input/Output*

On entry: A(i) must contain the x-component of the ith data point, x_i, for $i = 1, 2, \ldots, n+1$.

On exit: A(i) contains the value $x_i - x$, for $i = 1, 2, \ldots, n+1$.

2: B(N1) – ***real*** array *Input/Output*

On entry: B(i) must contain the y-component (function value) of the ith data point, y_i, for $i = 1, 2, \ldots, n+1$.

On exit: the contents of B are unspecified.

3: C(N2) – ***real*** array *Output*

On exit:

C$(1), \ldots,$ C(n) contain the first set of linear interpolations,

C$(n+1), \ldots,$ C$(2 \times n - 1)$ contain the second set of linear interpolations

$\vdots$

C$(n \times (n+1)/2)$ contains the interpolated function value at the point x.

4: N1 – INTEGER *Input*

On entry: the value $n+1$ where n is the number of intervals; that is, N1 is the number of data points.

5: N2 – INTEGER *Input*

On entry: the value $n \times (n+1)/2$ where n is the number of intervals.

6: N – INTEGER *Input*

On entry: the number of intervals which are to be used in interpolating the value at x; that is, there are $n+1$ data points (x_i, y_i).

7: X – ***real*** *Input*

On entry: the point x at which the interpolation is required.

6 Error Indicators and Warnings

None.

7 Accuracy

An estimate of the accuracy of the result can be made from a comparison of the final result and the previous interpolates, given in the array C. In particular, the first interpolate in the ith set, for $i = 1, 2, \ldots, n$, is the value at x of the polynomial interpolating the first $(i+1)$ data points. It is given in position $1 + \frac{1}{2}(i-1)(2n-i+2)$ of the array C. Ideally, providing n is large enough, this set of n interpolates should exhibit convergence to the final value, the difference between one interpolate and the next settling down to a roughly constant magnitude (but with varying sign). This magnitude indicates the size of the error (any subsequent increase meaning that the value of n is too high). Better convergence will be obtained if the data points are supplied, not in their natural order, but ordered so that the first i data points give good coverage of the neighbourhood of x, for all i. To this end, the following ordering is recommended as widely suitable: first the point nearest to x, then the nearest point on the opposite side of x, followed by the remaining points in increasing order of their distance from x, that is of $|x_r - x|$. With this modification the Aitken method will generally perform better than the related method of Neville, which is often given in the literature as superior to that of Aitken.

8 Further Comments

The computation time for interpolation at any point x is proportional to $n \times (n+1)/2$.

9 Example

To interpolate at $x = 0.28$ the function value of a curve defined by the points

$$\begin{pmatrix} x_i & -1.00 & -0.50 & 0.00 & 0.50 & 1.00 & 1.50 \\ y_i & 0.00 & -0.53 & -1.00 & -0.46 & 2.00 & 11.09 \end{pmatrix}.$$

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     E01AAF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NMAX, N1MAX, N2MAX
      PARAMETER        (NMAX=9,N1MAX=NMAX+1,N2MAX=NMAX*N1MAX/2)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X
      INTEGER          I, J, K, N
*     .. Local Arrays ..
      real             A(N1MAX), B(N1MAX), C(N2MAX)
*     .. External Subroutines ..
```

```
      EXTERNAL         E01AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'E01AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, X
      IF (N.GT.0 .AND. N.LE.NMAX) THEN
         READ (NIN,*) (A(I),I=1,N+1)
         READ (NIN,*) (B(I),I=1,N+1)
*
         CALL E01AAF(A,B,C,N+1,N*(N+1)/2,N,X)
*
         K = 1
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'Interpolated values'
         DO 20 I = 1, N - 1
            WRITE (NOUT,99999) (C(J),J=K,K+N-I)
            K = K + N - I + 1
   20    CONTINUE
         WRITE (NOUT,*)
         WRITE (NOUT,99998) 'Interpolation point = ', X
         WRITE (NOUT,*)
         WRITE (NOUT,99998) 'Function value at interpolation point = ',
     +     C(N*(N+1)/2)
      END IF
      STOP
*
99999 FORMAT (1X,6F12.5)
99998 FORMAT (1X,A,F12.5)
      END
```

9.2 Program Data

```
E01AAF Example Program Data
  5     0.28
    -1.00   -0.50    0.00    0.50    1.00    1.50
     0.00   -0.53   -1.00   -0.46    2.00   11.09
```

9.3 Program Results

```
 E01AAF Example Program Results

 Interpolated values
     -1.35680    -1.28000    -0.39253     1.28000     5.67808
     -1.23699    -0.60467     0.01434     1.38680
     -0.88289    -0.88662    -0.74722
     -0.88125    -0.91274

 Interpolation point =      0.28000

 Function value at interpolation point =     -0.83591
```